Calculus Midterm Topics

Limits and Continuity

A. Tangent lines (1.1)
B. Concept of limit (1.2)
   a. Definition of **Limit**
C. Computations of limits (1.3)
   a. Graphically
   b. Numerically
      i. One-sided
      ii. Two-sided
         1. From the +
         2. From the –
   c. **RULES**
      i. $\lim_{x \to a} c = c$
      ii. $\lim_{x \to a} x = a$
      iii. $\lim_{x \to a} [c * f(x)] = c * \lim_{x \to a} f(x)$
      iv. $\lim_{x \to a} [f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$
      v. $\lim_{x \to a} [f(x) * g(x)] = \lim_{x \to a} f(x) * \lim_{x \to a} g(x)$
      vi. $\lim_{x \to a} \lim_{x \to a}(\frac{f(x)}{g(x)}) = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$ (if, $\lim_{x \to a} g(x) \neq 0$)
      vii. Polynomial s
         1. $\lim_{x \to a}$ **p(x) = p(a)**
      viii. Limits DO NOT Exist when:
         - Unbounded
         - Oscillating
         - $\lim_{x \to a^+} \neq \lim_{x \to a^-}$
   d. Involving infinity (1.5)
      i. $\lim_{x \to \pm\infty} f(x) = \frac{\#}{0} \to \pm\infty$
      ii. $\lim_{x \to \pm\infty} f(x) = \frac{0}{\#} \to 0$
      iii. $\lim_{x \to \pm\infty} f(x) = \frac{\#}{\infty} \to 0$
      iv. Asymptotes
      v. End behavior
   e. Formal definition of limit (1.6)

D. Continuity (1.4)
   a. Formal definition using limits
      i. Continuous Functions
         1. Definition of **Continuous**

a. **f(a) is defined**
b. **$\lim_{x \to a}$ f(x) exists**
c. **$\lim_{x \to a}$ f(x) = f(a**

2. **RULES (f and g are continuous at x = a)**
   a. (f ± g) is continuous at x = a
   b. (f ∗ g) is continuous at x = a
   c. (f / g) is continuous at x = a, if g(a) ≠ 0

ii. Discontinuous Functions
1. **A Function is NOT Continuous at x=a when:**
   a. Hole in the graph → (f(a) is undefined)
   b. Redefined value for f(a) → ($\lim_{x \to a}$ f(x) ≠ f(a)
   c. Jump in the graph → ($\lim_{x \to a}$ f(x) does not exist)
   d. Unbounded → ($\lim_{x \to a}$ f(x) does not exist)

The Derivative

A. Definition of the derivative (2.1)
   a. Instantaneous rate of change
   b. Limit definition
      i. $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

B. Computations of derivatives
   a. Power rule (2.3)
   b. Product rule (2.4)
   c. Quotient rule (2.4)
   d. Trigonometric functions (2.6)
   e. Chain rule (2.5)
   f. Implicit differentiation (2.8)
   g. List of rules for your reference:
      - $\frac{d}{dx}c = 0$
      - $\frac{d}{dx}x = 1$
      - $\frac{d}{dx}x^n = nx^{n-1}$
      - $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$
      - $\frac{d}{dx}[cf(x)] = c f'(x)$
      - $\frac{d}{dx}[f(x)g(x)] = f'(x) g(x) + f(x) g'(x)$
      - $\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$

- $\frac{d}{dx}\sin x = \cos x$
- $\frac{d}{dx}\cos x = -\sin x$
- $\frac{d}{dx}a^x = a^x \ln a$
- $\frac{d}{dx}(\ln x) = \frac{1}{x}$

Applications of the Derivative

A. Linear Approximations (3.1)
B. Related rates (3.8)
C. Optimization (3.7)
D. L'Hopital's rule (3.2)
E. Graphing (3.6)
    a. Increasing/Decreasing (3.4)
        i. Mean Value Theorem (2.9)
        ii. Rolle's Theorem (2.9)
    b. Critical Points
    c. Concavity (3.5)
    d. Inflection points (3.5)
    e. Extrema
        i. Local (relative) Extrema
        ii. Global (absolute) Extrema
F. Physics (3.9)
    a. Position
    b. Velocity
    c. Acceleration (2.3)